Prediction of Drag Coefficient for Accelerated Single Sphere in Power Law Fluids

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Abstract

In the present study a single sphere accelerated in Polyacrylamide solution with different concentration (0.01, 0.03, 0.05 and 0.07) wt% as non Newtonian fluid within the considered range of power law index (0.6 – 1) and water as Newtonian fluid. Different types and sizes of spheres (stainless steel, glass and plastic) were used. Reynolds number (Re) and generated Reynolds number (Regn) varied between (1-888). Various equations for drag coefficient of a single sphere falling through Newtonian and non Newtonian fluid have been tested for different sizes and densities .It was found that a very similar Re–CD behavior. It is also clear that the most accurate correlation is the one proposed by Lali giving a 0.001619 RMS_ CD value. Archimedes' number was calculated for sphere motion at accelerated velocity and when it reached it's terminal velocity, and it was found that there was an uncertainty relation between Ar and Regn (R2= 0.572) for accelerated motion but there is a good relation between Ar and Regn (R2= 0.935) for sphere at its terminal velocity. The predicted values of Regn depended on Ar and showed a good relationship with the experimental values (R2= 0.958) as well as the measured drag coefficient.

Key words: **Single sphere; Power law fluid; Drag coefficient; Archimedes' number**.

1-Introduction

Overhead cranes are widely used in various industrial fields for being inexpensive and simple to install and easy to maintain. Designers seek to improve the performance of these cranes by increasing the speed of loading, transport and unloading.

This process is accompanied by the appearance of payload swing. This swing is an undesirable phenomena and is caused by using light flexible hoisting ropes as a part of the structure to reduce system mass; [9] [2]

. This phenomenon will lead to:

-1Low efficiency in transport and increase in the time of transport, because the crane operator has to wait until the vibration payload is finished in order to put the payload in the desired place.

-2The swing which could cause the hosting rope to leave its groove, leading to over wrapping, damage and safety accidents [2]

The existence of reliable mathematical model cranes helps the designers to design the parts of cranes perfectly to ensure high efficiency and safety as well as reducing manufacturing costs and time, especially, when implementing control system with such cranes.

The issue of the design of the crane theoretically to determine its performance depends on the amount of mass of the trolley, its speed, the forces acting on the trolley and the length of the rope and the mass of suspended payload in addition to the type and length of rail. Therefore, all these elements must be taken into consideration when studying the dynamics of cranes. The overall dynamic behavior of the crane is the result of the influence of dynamics of these elements and the mutual influence of these elements on each other.

Most published models in crane field take the crane as rigid body and neglect the presence of rail which leads to absence of the structure-trolley of crane interaction [9] [2], [1], [3] and [12]. The neglecting of the interaction in these models may be a source of error in mathematical model of a crane .

On the other hand, the structure-trolley interaction in a crane may be taken into consideration when the concept of moving load technique is applied to model the cranes. In general, two models of cranes based on moving load concept had been derived and used in previous papers.

Model-1, applies the moving load concept to cranes modeling. In this concept, the sum of the masses of the trolley and the payload are is modeled as one lumped mass which moves on the beam (rail) as shown in **Fig.1**; [18], [5],[6] . The state-variables of trolley position and swing angle are neglected. The effects of the values velocities and sometimes the accelerations of moving mass on dynamic response of rail (beam) are investigated in this approach. The values of velocities and accelerations of moving mass are given and assumed constant or dependent on mathematical function regardless of the driving forces, i.e. kinetics analysis of trolley.

Model-2; is the same as the one mentioned above, but the swing angle of the payload is taken as state-variable ;[19] , [13] ,[14] , and the kinetics analysis is still of trolley as shown in Fig.1. The effect of trolley motion on the swing angle can be investigated .

The crane performances are linked to the existence of the control systems which are called anti-swing control. These control systems generate a controlled force profile acting on the crane trolley to create the desired crane dynamics which is able to move the trolley adequately fast and to suppress the payload swing at the final position. Therefore; the study of the dynamic response systems when they are the driving forces is very important because of the large and obvious influence of the presence of these forces on changing the shape and pattern of dynamic responses of a system. In the case of cancellation of those forces, these effects cannot be observed

Table 1 Drag coefficient correlations (Clift et al., 1978)

Reynolds number range	Correlation
$Re \le 0.01$	$C_D = \frac{3}{16} + \frac{24}{\mathrm{Re}}$
$0.01 < Re \le 20$	$C_D = \frac{24}{\mathrm{Re}}\left[1 + 0.1315\,\mathrm{Re}^{(0.82-0.05W)}\right]$
$20 \le Re \le 260$	$C_D = \frac{24}{\mathrm{Re}}\left[1 + 0.1935\,\mathrm{Re}^{0.6305}\right]$
$260 \le Re \le 1.5\times10^3$	$\log C_D = 1.6435 - 1.1242W + 0.1558W^2$
$1.5\times10^3 \le Re \le 1.2\times10^4$	$\log C_D = -2.4571 + 2.5558W - 0.9295W^2 + 0.1049W^3$
$1.2\times10^4 \le Re \le 4.4\times10^4$	$\log C_D = -1.9181 + 0.6370W - 0.0636W^2$
$4.4\times10^4 \le Re \le 3.38\times10^5$	$\log C_D = -4.3390 + 1.5809W - 0.1546W^2$
$3.38\times10^5 \le Re \le 4.0\times10^5$	$C_D = 29.78 - 5.3W$
$4.0\times10^5 \le Re \le 10^6$	$C_D = 0.1W - 0.49$
$Re > 10^6$	$C_D = 0.19 - \left(8\times10^4/\mathrm{Re}\right)$

The most common approach taken by previous investigators is through the use of standard Newtonian relationships ($C_D - \mathrm{Re}$) but using a modified non Newtonian or generalized Reynolds number (Re_{gn}) (Kelessidis, 2003).

$$\mathrm{Re}_{gn} = \frac{\rho \upsilon^{2-n} d^n}{k} \qquad (3)$$

Kahn and Richardson in 1987 were tested various correlations proposed either for Newtonian or for non Newtonian fluids, it was found that the Newtonian curve, with $\mathrm{Re} = \mathrm{Re}_{gn}$, proposed elsewhere yielded quite accurate results for the non Newtonian fluids tested (18 source)

$$C_D = \left[2.25 R_e^{-.031} + 0.36 R_e^{0.06}\right]^{3.45} \quad (4)$$

$1 \le Re_{gn} \le 1000$

Miura et al (2001) extended the CD which was founded by Moleurs in 1993 for Newtonian fluids to non Newtonian fluid by assuming that $Re = Re_{gn}$,a reasonable agreement was found suggesting that the assumption made was acceptable for most engineering purposes.

$$C_D = \frac{24}{R_e} + \frac{4}{\sqrt{R_e}} + 0.4 \qquad Re < 10^5 \qquad (5)$$

Kelessidis and Mpandelis (2004) predicted the following drag coefficient correlation for power law fluid using nonlinear regression, for the total of 80 points,

$$C_D = \frac{24}{Re_{gn}}\left[1 + 0.1466 Re_{gn}^{0,378}\right] + \frac{0.44}{1 + 0.2635/Re_{gn}}$$

$0.1 < Re_{gn} < 1000$ (6)

Then they combined non-Newtonian data with Newtonian data, from their work and work from other investigators, giving a database of 148 pairs, an improved equation is derived.

$$C_D = \frac{24}{Re_{gn}}\left[1 + 0.1407 Re_{gn}^{0,6018}\right] + \frac{0.2118}{1 + 0.4215 / Re_{gn}}$$

$0.1 < Re_{gn} < 1000$ (7)

Lali et al, (1989) used five different carboxymethycellulose (CMC) solutions covering the range of power law fluid (n) between (0.555 – 0.85), with different diameter of glass beads and steel balls, thus covering Regn (0.1–200),the data correlated very well with the Newtonian curve defined by:

$$C_D = \frac{24}{R_e}\left[1 + 0.15 R_e^{0.687}\right]$$

$0.1 < Re < 1000$ (8)

Efforts were made to find empirical expressions for predicting the CD of varying degrees of complexity accuracy and with a reasonable compromise, which is attributed to Dallavalle equation. Felice (1999).

$$C_D = \left[\frac{4.8}{Re^{0.5}} + 0.63\right]^2 \qquad (9)$$

Cheremisinoff and Gupta (1983) proposed, a three part expression as the following :

$$C_D = \frac{24}{Re} \qquad Re < 3$$

$$C_D = \frac{24}{Re_{gn}} + \frac{4}{Re^{1/3}} \qquad 3 < Re < 500$$

$$C_D = 0.44 \qquad Re > 500 \qquad (10)$$

Turton and Clark (1987) proposed an explicit relationship, where they defined a dimensionless particle diameter, d_*,

$$d_* = \left[\frac{3 C_D Re^2}{4}\right]^{1/3} = d\left[\frac{\rho_f\left(\rho_s - \rho_f\right)g}{\mu^2}\right]^{1/3} \quad (11)$$

It should be noted that this equation can be recast as:

$$d_*^3 = \left[\frac{d^3 g\rho\Delta\rho}{\mu^2}\right] = \frac{\Delta\rho}{\rho}\frac{g\rho^2 d^3}{\mu^2} = \frac{\Delta\rho}{\rho} Ga = Ar \quad (12)$$

where Ga is Galileo number .

Koziol and Glowacki (1988) extended the approach of Schiller and Nauman to non-Newtonian power law fluids by forming the parameter

$$A = C_D^{2-n}\,\text{Re}^2 = \left(\frac{4}{3}\Delta\rho\right)^{2-n} \frac{d^{2+n} g^{2-n} \rho^2}{k^2} \quad (13)$$

This is independent of velocity. They presented a general plot of Re vs A based on their own data as well as data from other investigators. The data and plot covered the range of 0.001<Re<10.

Chhabra and Peri (1991) extended the approach of Koziol and Glowacki (1988) to higher Re, seeking a relationship of the form $Re = f\,(Ar, n)$, they defined the non-Newtonian Ar as

$$Ar = CD.\text{Re}^{\frac{2}{2-n}} = \frac{4gd^{2+n/2-n}}{3k^{2/2-n}.\rho^{2/n-2}}\left(\frac{\nabla\rho}{\rho}\right) \quad (14)$$

They gathered 400 experimental data points pertaining to $\text{Re} \geq 1$ and derived an equation, by minimizing the RMS error in velocity, in the form of

$$\text{Re} = aAr^b$$

Where;

$$a = 0.1 \times \exp\left(\frac{0.51}{n} - 0.73n\right) \quad (15)$$

$$b = \frac{0.954}{n} - 0.16$$

Their equation covered the range $10 < Ar < 10^6$ and $1 < Re < 10^4$.

Kelessidis (2004) found that the particle Re can then be expressed in terms of Ar and n as

$$\text{Re}_{gn} = aAr^b$$

Where ; $a = 0.01 \times \exp\left(\frac{1.42}{n}\right)$

$$b = \frac{0.881}{n} - 0.16 \quad (16)$$

The objective of this work is to collect experimental data of falling accelerated single sphere of different sizes and densities in Newtonian and Non-Newtonian type power law fluid. It is intended also, to improve the well known published correlations for C_D that have been in use, based on the collected data from many previous investigations.

2. Experimental Work

The experimental apparatus (Fig.1), consisted of a borosilicate glass cylindrical column of length 2.0 m and diameter of 0.3m.

Different spheres were used in the experiments as shown in Table 2. Characteristics of test fluids are shown in Table 3. A fishing string of 0.18 mm diameter, passed over an aluminum pulley to a drive weight that provided the driving force, suspended the sphere.

The external friction was reduced to a minimum with ball bearings on the pulley shaft. The sphere was submerged in the liquid of the column at an initial position of approximately 0.5m from the bottom. Upon release of the string the sphere rose under the action of falling weights. Measurement of the velocity of the sphere was carried out for different sphere diameters.

At the top of the column there was a system of light source and a photo-cell. A small pieces of eight light blocks were fixed on the part of the string that was un-submerged. As the sphere moved in the liquid, the light blocks also moved up through the collimator, which made the light intensity seen by the photo-cell varied and hence its resistance. This causes a variable voltage drop across the photo-cell.

An electronic circuit was constructed to measure the time elapsed between two successive light blocks. The electronic circuit components consist of light source, photo-cell detector and an interface unit connected to a personal computer.

The interface unit fed the response to the computer until all light blocks were passed. The velocity of the sphere versus time will be printed on the computer screen when a sphere submerges in the liquid column was accelerated under the action of a falling weight. Each run was repeated more than ten times to ensure experimental repeatability. The interval period between any two runs was approximately 10 minutes, this was the time necessary to collect the spheres and for the fluid to calm and release any bubbles that were created when the spheres were falling. After that Re and Re_{gn} was calculated, it was ranged (1- 888).

Table 2 Properties of spheres

Type	Diameter(m)	Density (kg/m³)
Stainless steel	0.01,0.012, 0.134 and 0.016	7660.22
Glass	0.072,0.093,0.0204 and 0.0256	2520
Plastic	0.0114 and 0.0138	1355

Table 3 properties of tested fluid

Type		Density (kg/m³)	Viscosity or Apparent viscosity (c.p)	Flow behavior Index ,n	Consistency Index k (Pa.sn)
PAA wt%	0.01	1000.2	2.05	0.834	0.01267
	0.03	1000.5	2.2	0.759	0.0229

	0.05	1000.7	2.25	0.684	0.0393
	0.07	1001.01	2.4	0.610	0.07
Tap water		1000	1		

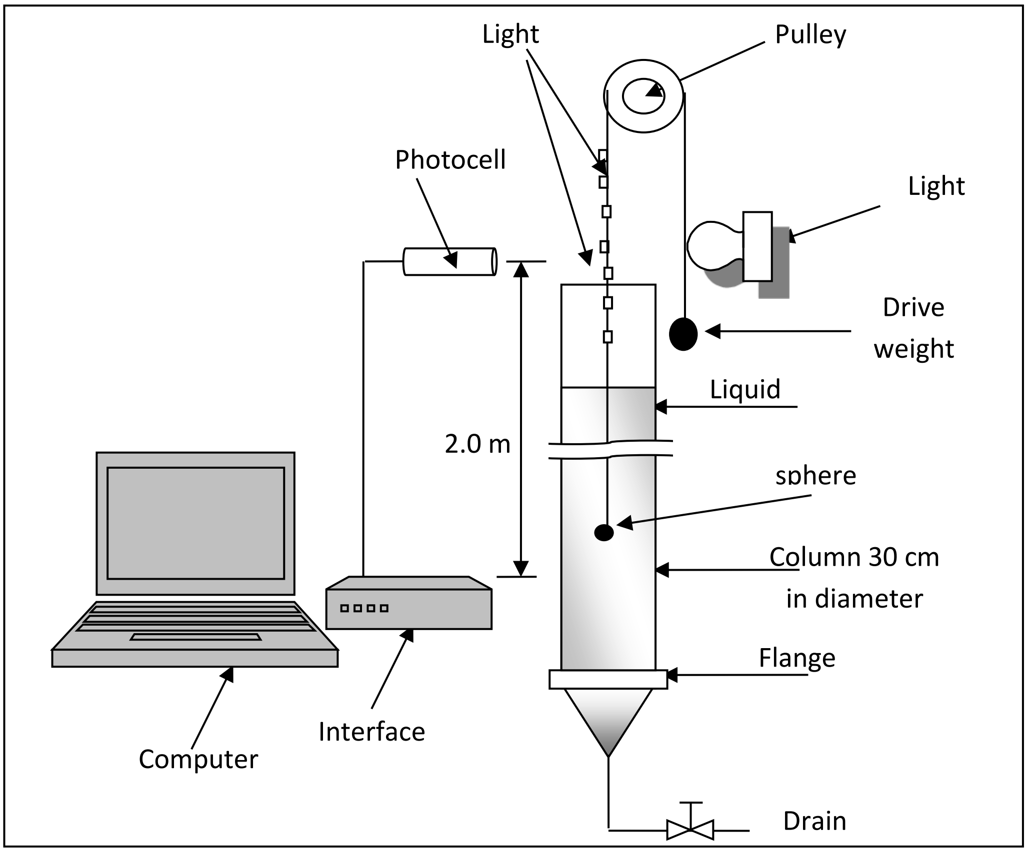

Fig. 1: The experimental apparatus

3. Results and Discussion

In order to calculate C_D for all 400 Newtonian and non Newtonian data available, different correlations proposed either for Newtonian or for non-Newtonian fluids were chosen, (Fig.2). It is apparent that the majority of the investigators conclude that the use of Newtonian correlations for non-Newtonian fluids is justified, with an engineering accuracy, provided that the apparent viscosity is used.

It can be seen from Fig.2 that the correlations in terms of the Re – C_D relationship are compared in a log – log graph. It is evident that the match is good for almost all correlations.

For finding the most suitable correlation, the RMS value is used, given as the differences of the C_D predicted by Clift and of the other correlations, since the correlation by Clift is recognized as the most accurate correlation.

This RMS value is, (Turton and Levenspiel, 1986).

$$RMS - C_D = \sqrt{Q_{C_D} / N} \qquad (17)$$

Where

$$Q_{C_D} = SUM\left(\log_{10}(C_D) - \log_{10}(C_{D_i})\right)^2 \qquad (18)$$

Where C_D is the drag coefficient predicted from Clift and CD_i is the drag coefficient predicted from the other stated correlations, Table. 4.

In table 4 the different indicators, RMS- C_D are compared, computed as stated above, it shows that the correlations compare favorably with the most accurate but cumbersome correlation Clift. It is clear that all RMS is convergent. It is also apparent that the most accurate is the one proposed by Lali giving an RMS value for the difference in the logarithms of C_D between their proposal and that of Clift of only 0.001619. For all the 400 data points of Newtonian and Power law fluid. Hence the correlation proposed by Lali will be used in this study to calculate the C_D for the power law fluid.

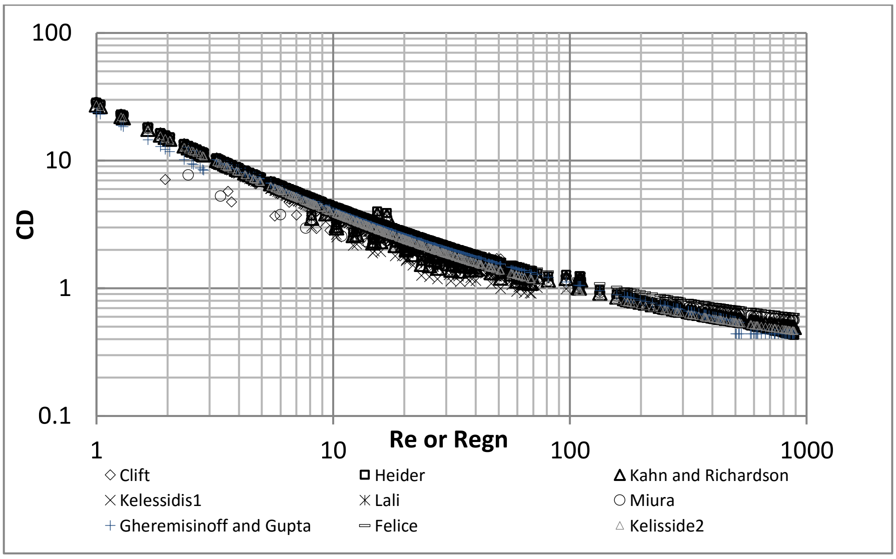

Fig. 2 :Log – Log graph of C_D – Re from various correlations

Table 4: RMS for drag coefficient (10^{-3})

Ref.	**Heider**	**Kahn and Richardson**	**Kelessidis**	**Lali**	**Miura**	**Cheremisinoff & Gupta**	**Filice**	**Kelessidis and Mpandelis**
RMS	1.752	1.6599	6.874	1.619	3.454	3.1658	6.190	2.879

3.1Archimedes’ number (Ar)

In order to verify the applicability of (Ar) for accelerated sphere in power law fluid, (Ar) for sphere as global state (accelerated and terminal velocity) was calculated, its values ranged between (9.17- 756), then plotted with Re_{gn} in log – log scale as seen in Fig.(3), from this figure it can be seen that there is uncertainty relation between Ar and Re_{gn} (R^2= 0.572), but when Re_{gn} is plotted vs. Ar for falling sphere at its terminal velocity, Fig. (4), it can be seen that there is a good relation between Ar and Re_{gn} (R^2= 0.935) then for sphere at terminal velocity the Re_{gn} can be expressed in terms of Ar and power law index (n) by using Eq. (16).

So with the same approach propose a similar expression to evaluate experimental data for sphere at its terminal velocity. Suggested simple empirical equation describes the relation between Ar and Re_{gn} for non-Newtonian fluids

$$Re_{gn} = aAr^{b} \qquad (19)$$

Where $a = 0.01 \times \exp\left(\frac{1.5}{n}\right)$

$$b = \frac{1.126}{n} - 0.2$$

Covered the range of ;

$$9.1 \le Ar \le 455.5 \qquad 9.4 \le Re_{gn} \le 110$$

With R^2= 0.958 as can seen in the **Fig. 5**.

In order to show the effect of Re_{gn} predicted by Eq. (19) and C_D measured, Fig. (6) Show the relationship between predicted Re_{gn} and C_D measured. It can be seen that there is a good relation between the predicted data with the experimental data.

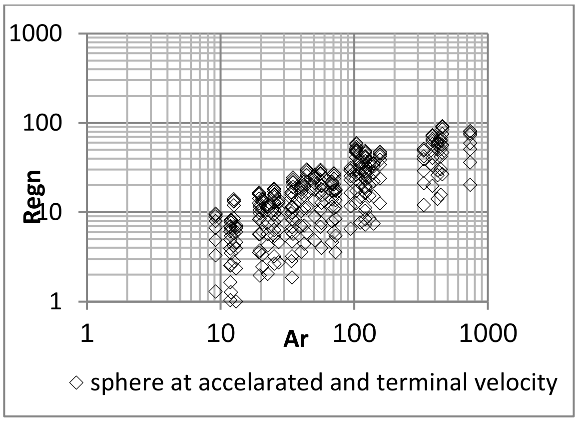

Fig 3: Archimedes’ number and generated Reynolds number for different sphere type as global state (accelerated and terminal velocity)

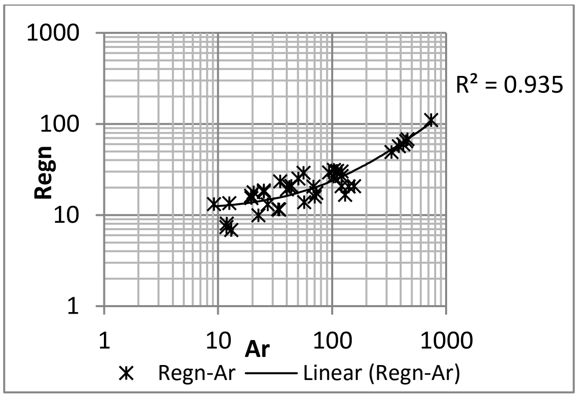

Fig 4: Archimedes’ number verse generated Reynolds number for different sphere at its terminal velocity

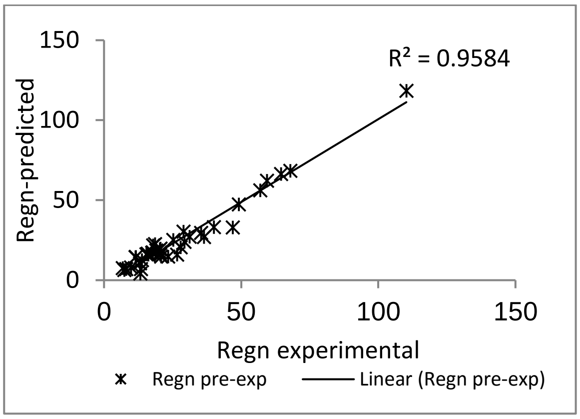

Fig5: Comparison between experimental generated Reynolds number and predicted

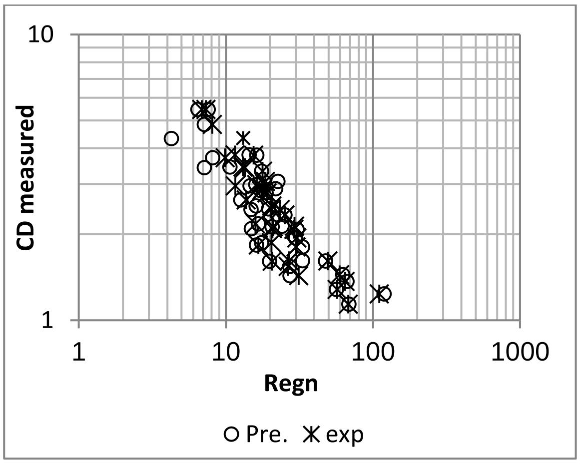

Fig 6: Show the relation between calculated drag force with experimental generated Reynolds number and with that calculated

4. Conclusions

Various equations for drag coefficient of sphere falling through Newtonian fluid have been testing for single sphere of different sizes and densities falling through Newtonian and non Newtonian fluid compared and yielded very similar Re–C_D behavior. It is also clear that the most accurate is the one proposed by Lali giving an RMS_C_D value of only 0.001619.

Archimedes' number for sphere as global state (accelerated and terminal velocity) was calculated and it be concluded that there is an uncertainty relation between Ar and Re_{gn} (R^2= 0.572) but there is a good relation between Ar and Re_{gn} (R^2= 0.935) for sphere at its terminal velocity.

The predict values of Re_{gn} depending on Ar show good relationship with the experimental values R^2= 0.958 and with the measured C_D.

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Nomenclature

Re	Reynolds number
Re_{gn}	generated Reynolds number
C_D	Drag coefficient
RMS	Root mean Square
Ar	Archimedes' number
d	Sphere diameter, m
v	Sphere velocity, m/s
n	Power law index (flow behavior index),dimension less
k	Consistency index, (Pa.sn)
μ	Viscosity, kg/m.s
ρ	Density of fluid, kg/m^3
d_*	dimensionless particle

	diameter
Ga	Galileo number

التنبؤ بمعامل الاحتكاك لكرة منفردة معجلة في سائل قانون الطاقة

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الخلاصة

تم في البحث دراسة حركة كرة واحدة في محلول بولي أكريمايد وبتراكيز مختلفة (0.01، 0.03، 0.05 و 0.07)٪ وزناً كسائل غير نيوتوني والماء كسائل نيوتيني. وباستخدام أنواع مختلفة من الكرات (الفولاذ المقاوم للصدأ والزجاج والبلاستيك) وباقطار وكثافات مختلفة.وتبين من هذه التجارب ان رقم رينولدز (Re_{gn}) تراوحت بين (1-888) ضمن مدى مؤشر الجريان تراوح ما بين (0.6-1)

تم اختبار مجموعة من معادلات معامل الاحتكاك لكرة تتحرك في سوائل نيوتينية وغير نيوتينية والمقارنة بينهم عن طريق حساب RMS وتبين ان افضل معادلة تحقق النتائج هي معادلة Lali حيث كانت قيمة MS_ CD 0.001619. كما تضمن البحث حساب رقم أرخميدس، وتبين انه ليس هناك علاقة بين رقم ارخميدس ورقم رينولدرز($R^2 = 0.572$) ولكن هناك علاقة جيدة بينهما (($R^2 = 0.935$ عندما تصل الى السرعة النهائية

الكلمات الرئيسية : كرة مفردة ; سائل قانون الطاقة ; معامل الاحتكاك ; رقم ارخميدس