

Investigation of Vertical and Pitch Road Vehicle Dynamic Responses to Improve the Critical Speed Using Controllable Semi-Active PID Suspensions

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Abstract:-

Vertical and pitch dynamic responses are investigated and examined to improve the critical speed of a road vehicle. The objective of this paper is to improve the road vehicle critical speed by using new design of semi-active PID suspension equipped with conical spring stiffness installed at front and rear wheels which has variable magnitudes of spring stiffness that will depress the oscillations effectively. A numerical full road vehicle simulation model comprising mathematical differential equations of operation with (9) degrees of freedom and passive PID suspensions with helical shaped spring is constructed. Mathematical differential equations of operation contain all rectilinear and angular displacements of a road vehicle such as vertical displacements of road vehicle carbody with the front and rear wheels. Also roll, pitch, yaw, and lateral dynamic responses are included in the mathematical differential equations while a special technique is used to transform these second order differential equations into first order to reduce the computational time. Computer-aided simulation model with Matlab Program is adopted at different speeds and different sudden step steer angles. In order to satisfy the reliability of the adopted road vehicle simulation model, an experimental work using Hyundai-Tucson 2009 is carried out to examine the lateral and vertical dynamic responses of the road vehicle carbody. In the present simulation model the critical speed of a road vehicle is improved by 13.4% using semi-active PID suspension which is equipped with conical shaped spring at front and rear wheels.

Keywords:- road vehicle dynamic; vertical response; pitch response; critical speed; semi-active PID suspension

1. Introduction

The road vehicle dynamic behavior against the introduced forces and moments at the tire-road contact area depends upon different conditions such as side slip angles, vertical tire load and the present road surface quality. Forces

and moments which are introduced due to the tire-road contact parameters will create undesirable vibrations and oscillations that are immediately transformed to the road vehicle carbody through a set of PID suspension elements in which these

vibrations in turn causing some prospective mechanical problems and influencing ride passenger comfort during road vehicle instability. The instability of a road vehicle during motion subjected to different vehicle speeds and different sudden step steer angles is existed according to the absent of a road vehicle control at a critical value of vehicle speed which is well-known as a critical speed.

This study is undertaken to improve the characteristics of the road vehicle handling by improving the critical speed by means of using suspensions with variable spring stiffness. Ride passenger comfort is the main goal for this research in which it is considered as the research question. Many studies and researches have been done in the last years to improve the critical speed of the road vehicle adopting investigation on different dynamical responses. In the study of Zuraulic V. et al [1], the parameters important for lateral dynamics of vehicles are analyzed in order to establish the values of the critical speed on the moment of losing the stability. It is concluded in the study of Akhilesh Kumar Maurya et al [2] that the speed at which maximum deceleration rate occurred (referred here as critical speed) depends on vehicle type and its maximum speed. Also it is observed that the critical speed (where deceleration is maximum) depends on

vehicle type. A full-vehicle active suspension system is designed to simultaneously improve vehicle ride comfort and steady-state handling performance in the study of Jun Wang et al [3]. In addition, this research presents that the most common measure of vehicle steady-state handling performance is the understeer gradient [4], by which vehicles are categorized into three types: neutral steer, understeer and oversteer. In the neutral steer case, the lateral acceleration at the gravity center of the vehicle will yield an identical increase in slip angle at both front and rear wheels. In the understeer case, the lateral acceleration will cause more front-wheel slip. An oversteer vehicle could lose its directional stability at the critical speed and vehicle understeer gradient varies due to transient maneuvers. In the present study the vertical and pitch road vehicle dynamic responses are investigated and analyzed to identify the critical speed of a road vehicle subjected to different vehicle velocities and sudden step steer angles with helical shaped spring of passive PID suspensions.

2. Theoretical Study

The characteristics of the handling performance of a Road vehicle are proportional directly to the transmitted forces and moments introduced horizontally and vertically at the tire-

road contact area as a result of steering, braking and driving operations as well as sudden external road disturbances as shown in **Figure 1**.

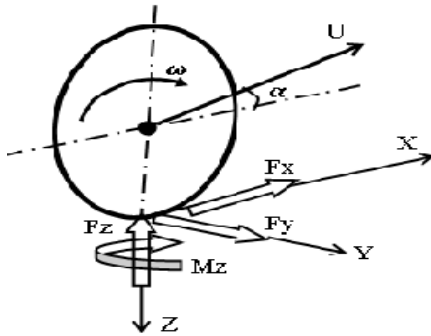


Fig.1 Forces and moments introduced at the tire-road contact area of a road vehicle
 The road vehicle motion undergoes unstable state when the critical speed is exceeded in which it could be happened at the oversteer situation of a road vehicle where it should be moved with speed less than the associated critical speed. In fact the directional behavior of a road vehicle during maneuver at different vehicle speeds is highly dependent upon the relationships between the side slip angles of the front and rear tires whereas the steady-state cornering characteristics could be classified into neutralsteer, understeer and oversteer as shown in **Figure 2**. In understeer situation the slip angles of the front tiers are more than that of the rear tires so we should turn the steering wheel more than expected to maintain the road vehicle on a desired path but conversely the slip angles of rear tires are more than that of front tires in

oversteer situation and we should turn the steering wheel less than expected. Neutral steering situation may be existed when the slip angles are equal for both front and rear tires

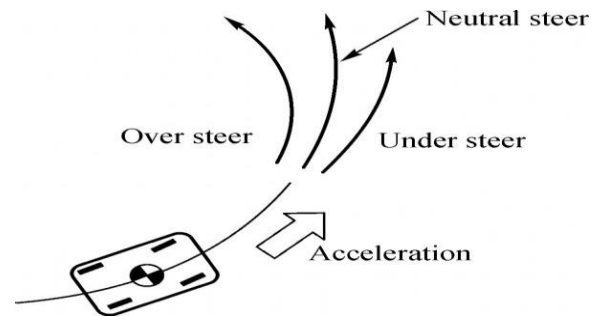


Fig. 2 Classification of the steady-state cornering characteristics

As shown in **Figure 3**, the steer angle rapidly decreases until it becomes zero at the value of the road vehicle critical speed where instability starts to be existed. In this figure it can be observed that the ratio (L/R) plays an important role in the classification of the road vehicle cornering characteristics where L is the wheel base and R is the cornering radius.

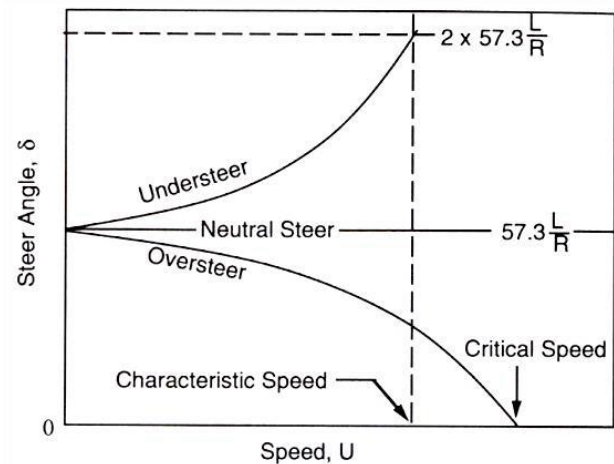


Fig. 3 Relationships between steer angles and speed of neutralsteer, understeer and oversteer vehicles

3. Mathematical Full Model of Road Vehicle

Road vehicle is mathematically modeled with (9) degrees of freedom including vertical displacement of the road vehicle carbody and for the front and rear wheels combining with roll, pitch, yaw and lateral movements of the road vehicle carbody as shown in **Figure 4**.

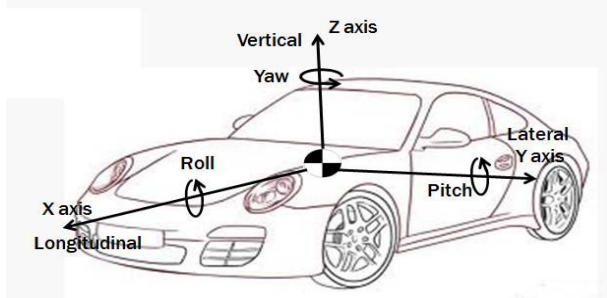


Fig. 4 Road vehicle displacements
Mathematical differential equations of operation for the full car road vehicle model

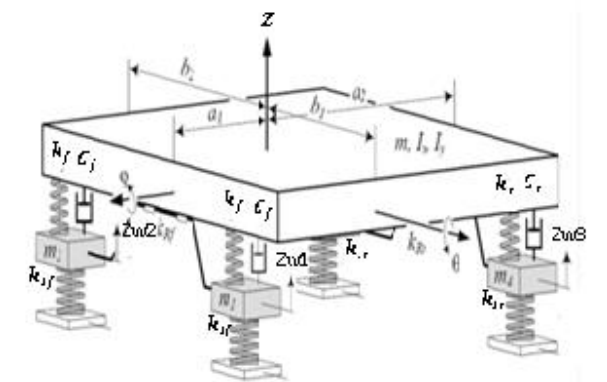


Fig.5 Full car road vehicle simulation model [5]

Expressing vertical, roll, and pitch are derived as in the following equations according to the road vehicle movements and displacements as shown in **Figure 5**. In which the full road vehicle car is modeled with 9 degrees of freedom including vertical, roll, pitch, yaw and lateral 5 equations for the road vehicle carbody while 4 equations are derived for the vertical motion of the front and rear wheels. The equation of the vertical displacement of carbody is [5],

$$\begin{aligned}
 m\ddot{Z}_c = & -2(C_f + C_r)\dot{Z}_c + C_f\dot{Z}_{w1} + \\
 & C_f\dot{Z}_{w2} + C_r\dot{Z}_{w3} + C_r\dot{Z}_{w4} + \\
 & (-b_1 + b_2)(C_f + C_r)\dot{\phi} + \\
 & 2(C_f a_1 - C_r l a_2)\dot{\theta} - 2(K_f + K_r)Z + \\
 & K_f Z_{w1} + K_f Z_{w2} + K_r Z_{w3} + K_r Z_{w4} + \\
 & (-b_1 + b_2)(K_f + K_r)\phi + \\
 & 2(K_f a_1 - K_r a_2)\theta
 \end{aligned} \tag{1}$$

While equation of the vertical displacements of front and rear wheels are [5],

$$\begin{aligned}
 m_w\ddot{Z}_{w1} = & C_f\dot{Z} - C_f\dot{Z}_{w1} + C_f b_1\dot{\phi} - \\
 & C_f a_1\dot{\theta} + K_f Z + \left(K_f - K_{rf} \frac{1}{(b_1+b_2)^2} - \right. \\
 & \left. K_{tf}\right) Z_{w1} + \left(K_f b_1 + K_{rf} \frac{1}{b_1+b_2}\right) \phi - \\
 & K_f a_1\theta + K_{rf} \frac{Z_{w2}}{(b_1+b_2)^2} + K_{tf} Z_{w1}
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 m_w\ddot{Z}_{w2} = & C_f\dot{Z} - C_f\dot{Z}_{w2} - C_f b_2\dot{\phi} - \\
 & C_f a_1\dot{\theta} + K_f Z - \left(K_f - K_{rf} \frac{1}{(b_1+b_2)^2} - \right. \\
 & \left. K_{tf}\right) Z_{w2} - \left(K_f b_2 + K_{rf} \frac{1}{b_1+b_2}\right) \phi -
 \end{aligned}$$

$$K_f a_1 \theta + K_{rf} \frac{Z_{w2}}{(b_1+b_2)^2} + K_{tf} Z_{w2} \quad (3)$$

$$m_w \ddot{Z}_{w3} = C_f \dot{Z} - C_f \dot{Z}_{w3} + C_r b_1 \dot{\phi} + C_r a_2 \dot{\theta} + K_r Z - \left(K_r + K_{rr} \frac{1}{(b_1+b_2)^2} + K_{tf} \right) Z_{w3} + \left(K_r b_1 + K_{rr} \frac{1}{b_1+b_2} \right) \phi + K_r a_2 \theta + K_{rr} \frac{Z_{w3}}{(b_1+b_2)^2} + K_{tr} Z_{w3} \quad (4)$$

$$m_w \ddot{Z}_{w4} = C_f \dot{Z} - C_f \dot{Z}_{w4} - C_r b_2 \dot{\phi} + C_r a_2 \dot{\theta} + K_r Z - \left(K_r + K_{rr} \frac{1}{(b_1+b_2)^2} + K_{tf} \right) Z_{w4} - \left(K_r b_2 + K_{rr} \frac{1}{b_1+b_2} \right) \phi + K_r a_2 \theta + K_{rr} \frac{Z_{w4}}{(b_1+b_2)^2} + K_{tr} Z_{w4} \quad (5)$$

The equation of roll angular displacements for the carbody are constructed as [5],

$$I_{xx} \ddot{\theta} = -[(b_1 - b_2)(C_f - C_r)] \dot{Z} + b_1 C_f \dot{Z}_{w1} - b_2 C_f \dot{Z}_{w2} - b_1 C_r \dot{Z}_{w3} + b_2 C_r \dot{Z}_{w4} - [(b_1^2 + b_2^2)(C_f - C_r)] \dot{\phi} + [(a_1 b_1 - a_1 b_2) C_f + (a_2 b_1 - a_2 b_2) C_r] \dot{\theta} - [(b_1 - b_2)(K_f - K_r)] Z + b_1 K_f Z_{w1} - b_2 K_f Z_{w2} - b_1 K_r Z_{w3} + b_2 K_r Z_{w4} - [(b_1^2 + b_2^2)(K_f - K_r)] \phi + [(a_1 b_1 - a_1 b_2) K_f + (a_2 b_1 - a_2 b_2) K_r] \theta \quad (6)$$

Whereas the equation of pitch angular displacements for the carbody are constructed as [5],

$$I_{yy} \ddot{\theta} = 2(a_1 C_f - a_2 C_r) \dot{Z} - a_1 C_f \dot{Z}_{w1} - a_1 C_f \dot{Z}_{w2} + a_2 C_r \dot{Z}_{w3} + a_2 C_r \dot{Z}_{w4} + [(a_1 b_1 - a_1 b_2) C_f -$$

$$(a_2 b_1 - a_2 b_2) C_r] \dot{\phi} - 2(a_1^2 C_f + a_2^2 C_r) \dot{\theta} + 2(a_1 K_f - a_2 K_r) Z - a_1 K_f Z_{w1} - a_1 K_f Z_{w2} + a_2 K_r Z_{w3} + a_2 K_r Z_{w4} + [(a_1 b_1 - a_1 b_2) K_f - (a_2 b_1 - a_2 b_2) K_r] \phi - 2(a_1^2 K_f + a_2^2 K_r) \theta \quad (7)$$

Yaw and lateral motion for the carbody are constructed as the following equations as shown in **Figure 6**. Yaw motion of the carbody can be expressed as the following equation,

$$I_{zz} \ddot{\psi} = 2l_f C_f \delta \cos \delta - \left(\frac{2l_f C_f}{U} \cos \delta \right) \dot{Y} - \left(2l_f^2 \frac{C_f}{U} \cos \delta \right) \dot{\psi} + 2l_r \frac{C_r}{U} Y - 2l_r^2 \frac{C_r}{U} \dot{\psi} + 2\mu g l_f \left(\frac{m}{4} + m_w \right) \sin \delta \quad (8)$$

The lateral motion of the carbody is written as,

$$m \ddot{Y} = 2C_f \delta \cos \delta - \left(\frac{2C_f}{U} \cos \delta \right) \dot{Y} - \left(2 \frac{C_f}{U} l_f \cos \delta \right) \dot{\psi} - 2 \frac{C_r}{U} \dot{Y} + 2 \frac{C_r}{U} l_r \dot{\psi} + 2\mu g \left(\frac{m}{4} + m_w \right) \sin \delta + U \dot{\psi} \quad (9)$$

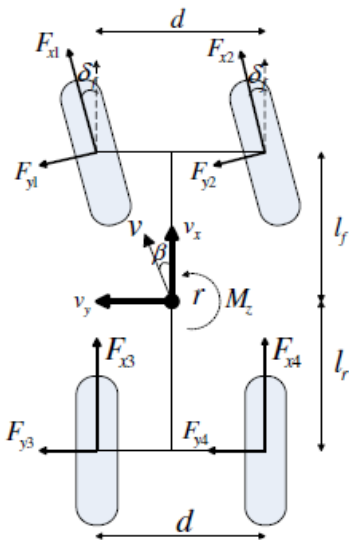


Fig.6 Road vehicle Yaw and lateral motion

4. Experimental Work

The road vehicle used for an experimentally work in this study is Hyundai Tucson 2009, which is a compact crossover produced by the South Korean manufacturer Hyundai. In order to measure the vehicle vibration experimentally, some specific locations are chosen for vibration sensors to investigate the displacement and velocity for different speeds and different sudden step steer angles. **Figure 7** shows the vibration meter and sensors located on the top, side and front of the vehicle. Some of the experimental results which are obtained by using semi-active road vehicle suspension controller are shown in **Figures 8-9**. In which the experimental results are very closer to the magnitudes obtained by the

full car simulation model provided with semi-active PID suspension controller.



Fig.7 Vibration meter and sensors used to investigate road vehicle vibrations

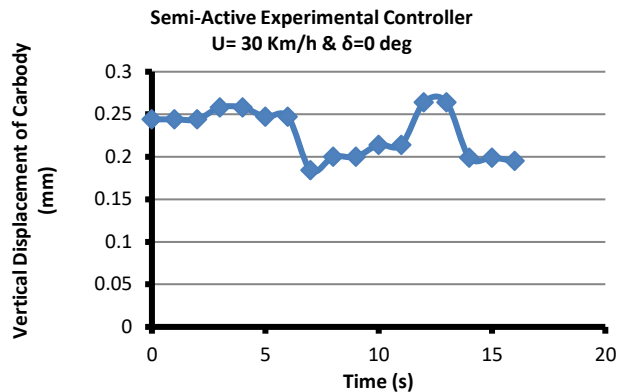


Fig.8 Road vehicle vertical experimentally dynamic response of the carbody when the vehicle is accelerated up to speed 30 Km/h with zero sudden step steer angle

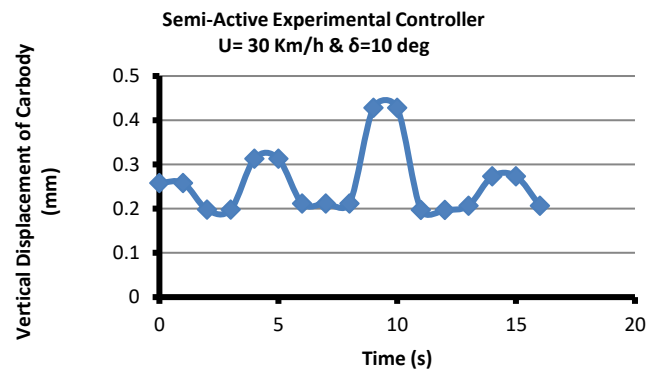


Fig. 9 Road vehicle vertical experimentally dynamic response of the carbody when the

vehicle is accelerated up to speed 30 Km/h with 10 deg. sudden step steer angle

5. Road Vehicle Simulation Model

Computer-aided simulation is presented to investigate the vertical and pitch dynamic responses of a road vehicle subjected to different vehicle velocities and sudden step steer angles in which the road vehicle is equipped with helical shaped spring of passive PID suspension controller. A special technique is used to transform the mathematical differential equations of operation for the full road vehicle into first order differential equations in order to be easily solved with computer program Matlab and to reduce the computational time [7]. **Figures** below show the simulation results of the vertical and pitch road vehicle dynamic responses in order to identify the value of the critical speed. When vertical and pitch dynamic responses are examined by the simulation model, it is observed that the vehicle speed 184 Km/h is considered as the value of the critical speed for the road vehicle subjected to zero sudden step steer angles as shown in **Figure 10**. Also it can be noticed from **Figure 11** that the critical speed of road vehicle occurs at 64 deg. Sudden step steer angle. As shown in **Figure 12** the vertical and Pitch dynamic responses of road vehicles

carbody subjected to the critical speed 184 Km/h and zero sudden step steer angles are adopted with passive and semi-active PID suspension controllers. It can be clearly observed that road vehicle with semi-active PID suspension controller is under the value of the road vehicle critical speed in contrast with passive PID suspension controller at the critical speed 184 Km/h. But in **Figure 13** it can be shown that the critical speed is improved by using semi-active PID suspension controller up to 210 Km/h in which it means that the critical speed of a road vehicle is improved in 13.4% by using semi-active PID suspension controller with conical shaped spring.

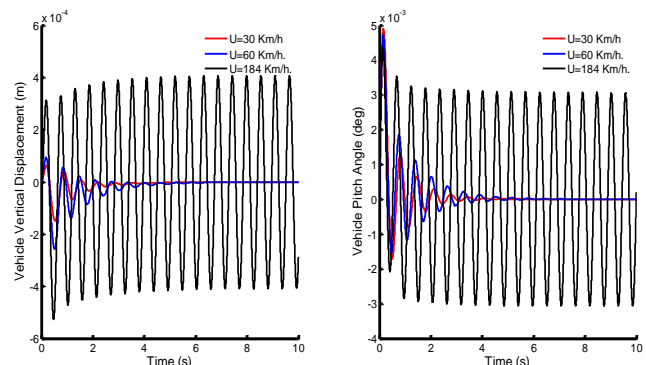


Fig.10 Vertical and pitch dynamic responses of a road vehicle carbody subjected to different vehicle speeds with zero sudden step steer angles equipped with helical shaped spring passive PID suspension controller.

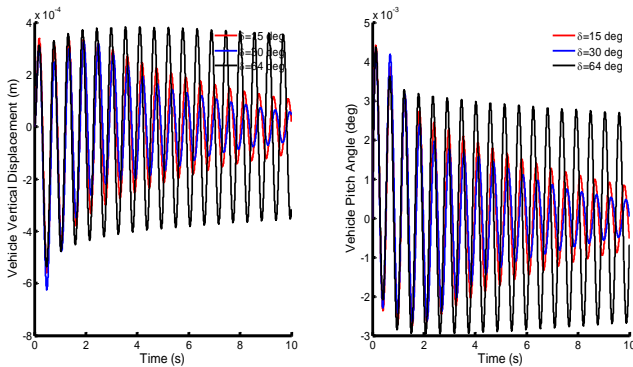


Fig.11 Vertical and pitch dynamic responses of a road vehicle carbody subjected to different sudden step steer angles and 184 Km/h speed equipped with helical shaped spring passive PID suspension controller.

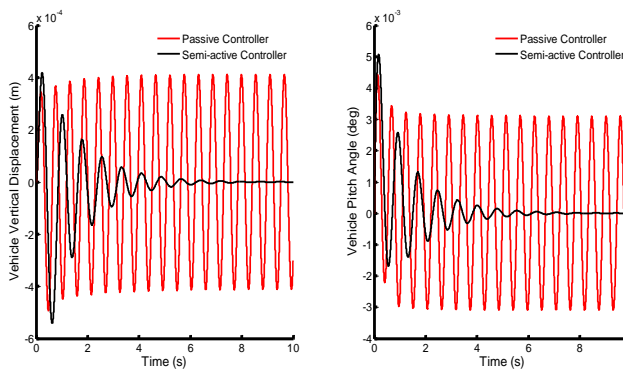


Fig.12 Vertical and Pitch dynamic responses of road vehicles carbody subjected to the vehicle critical speed 184 Km/h and to zero sudden step steer angles with passive and semi-active PID suspension controllers.

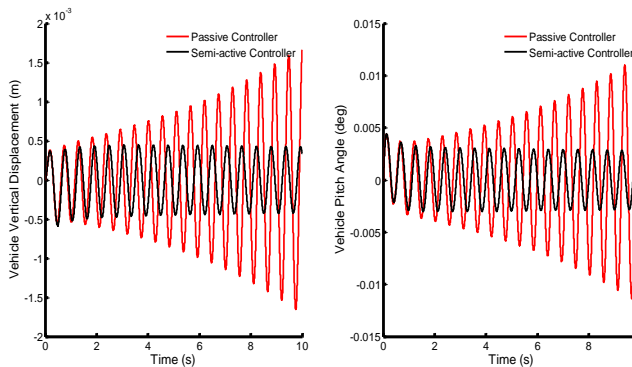


Fig.13 Vertical and Pitch dynamic responses of road vehicles carbody subjected to the vehicle improved critical speed 210 Km/h and to zero sudden step steer angles with passive and semi-active PID suspension controllers.

6. Conclusion

It is previously observed that the critical speed of a road vehicle subjected to different vehicle velocities and different sudden step steer angles is about 184 Km/h when the road vehicle using passive PID suspension controller with helical shaped spring of constant magnitude of stiffness. But this critical speed is improved and raised up to 210 Km/h by using semi-active PID suspension controller with conical shaped spring of variable magnitudes of stiffness. And it can be concluded that the critical speed of a road vehicle is improved about 13.4% using semi-active PID suspension controller.

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التحقق من استجابة المركبة للحركة العمودية والدورانية باستخدام المسيطر PID شبه النشاط

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الخلاصة:

تم في هذا البحث دراسة استجابة المركبة للحركة العمودية والدورانية والتحقق منها لتحسين السرعة الحرجة للمركبات التي تسير على الطرق. الهدف من هذا البحث هو لتحسين السرعة الحرجة باستخدام تصميم مسيطر PID شبه النشاط والمجهزة بمنظومة تعليق مخروطية مثبتة على العجلات الأمامية والخلفية والتي لها صلادته متغيره دورها ضغط التذبذب بشكل كبير. ابتداءا تم بناء نموذج رياضي لسيارة الطريق مع معادلات تفاضلية من الدرجة الثانية ولغاية 9 درجات حرية لحركة السيارة والعجلات الأمامية والخلفية. بالإضافة إلى ذلك، فإن العزوم

الثلاث حول المحاور الثلاث والمسماة (Roll, Pitch and Yaw) والحركة الجانبية للمعادلات التفاضلية هي مستمدة للتحقيق في السلوك الديناميكي لسيارة الطرق مع تعرضها لمعايير مختلفة: مثل سرعة مختلفة وزوايا استدارة مختلفة من أجل تحسين السرعة الحرجة للسيارة على الطريق حيث تم تحويل كل هذه المعادلات التفاضلية من الدرجة الثانية إلى معادلات تفاضلية من الدرجة الأولى باستخدام تقنية حاسوبية خاصة لتكون سهلة في التعامل مع البرامج الحاسوبية. تم اعتماد نموذج محاكاة بمساعدة الحاسوب مع برنامج الماتلاب ولسرعة مختلفة ولزوايا توجيه مفاجئة ومختلفة. وللتوثق من نموذج المحاكاة تم إجراء تجربته عملية على سيارة هونداي توكسون موديل 2009 لدراسة الاستجابات الديناميكية الجانبية والرأسيه لهيكل المركبة. من خلال نموذج المحاكاة تم تحسين السرعة الحرجة بنسبة وصلت الى 13.4% باستخدام نظام التعليق PID شبه النشط مجهز بناقض مخروطي الشكل في العجلات الأمامية والخلفية.

الكلمات الرئيسية: ديناميكية المركبات, الاستجابة العمودية, استجابة العزم, السرعة الحرجة, منظومة التخميد PID شبه النشط